

Ques. what do you understand by resolving power?

Explain Rayleigh's criterion for the limit of resolution. obtain an expression for the resolving power of a prism.

Ans. Resolving power \rightarrow The ability of an optical instrument to just resolve the images of two near by points sources. is called its resolving power.

Sunday

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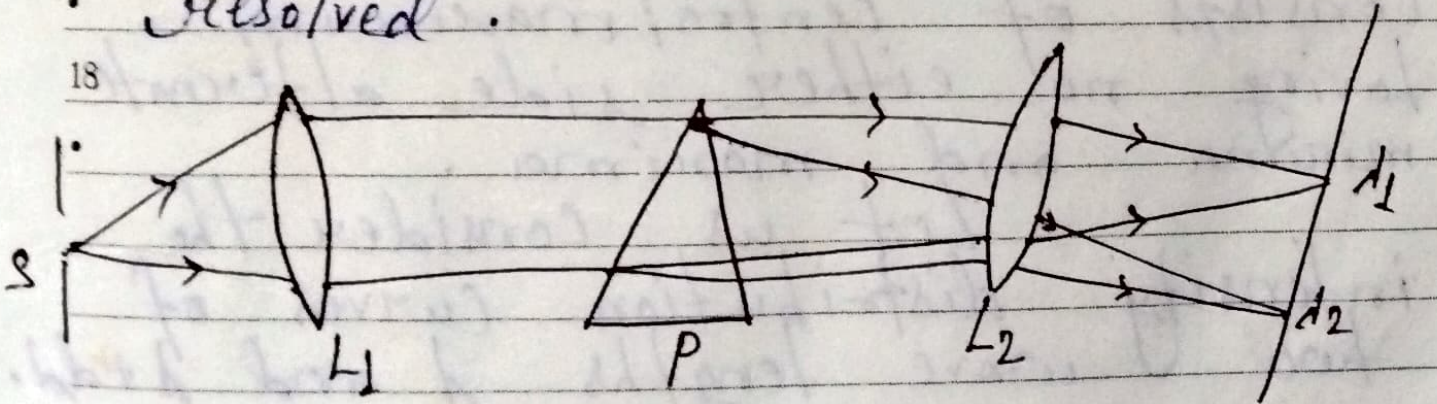
February

34th Day

Considering a simple prism spectrograph where S is narrow slit illuminated by a source which emits two close wavelengths λ_1 and λ_2 , a spectrum consisting of two lines corresponding to λ_1 and λ_2

is received in the focal plane of L_2 .

In reality the focus of the prism act as diffracting apertures. So the two lines in the spectrum are actually two Fraunhofer diffraction patterns close together having an intensity distribution. The two patterns overlaps each other in general but overlapping is only to a little extent, the principle maxima the two patterns are indistinguishable the lines are said to be resolved. On the other hand if patterns overlap to such extent that the resultant intensity indicates no dip in the middle then it is impossible to judge whether the pattern corresponds to a single line or two close lines. In such condition lines are not resolved.



Notations

Therefore, the resolving power of such an optical instrument is defined spectral lines.

Rayleigh's Criterion for resolution:

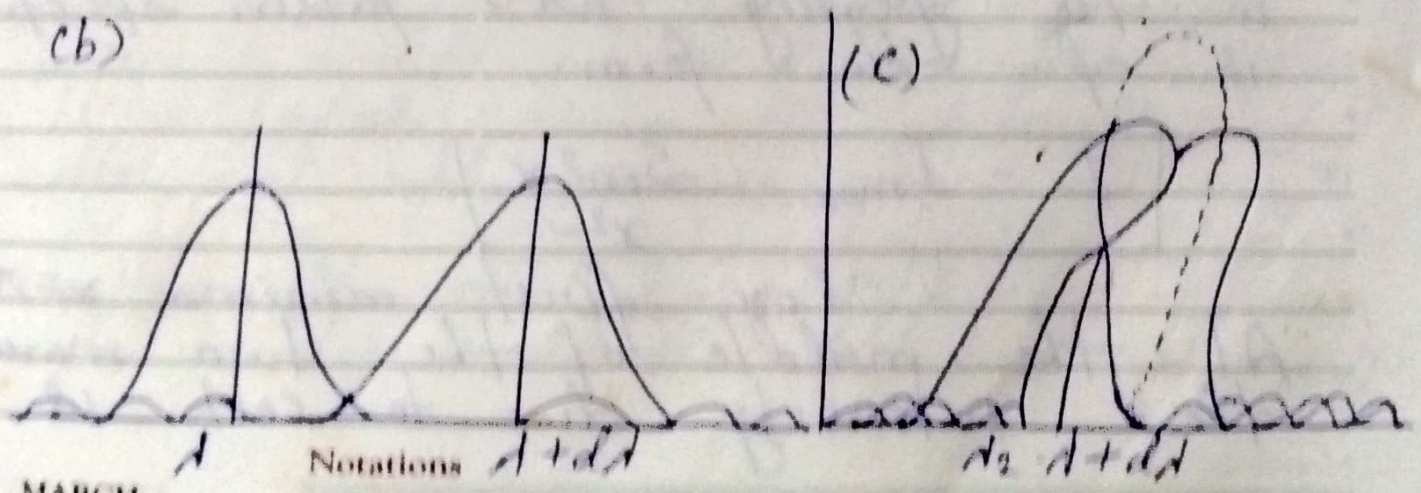
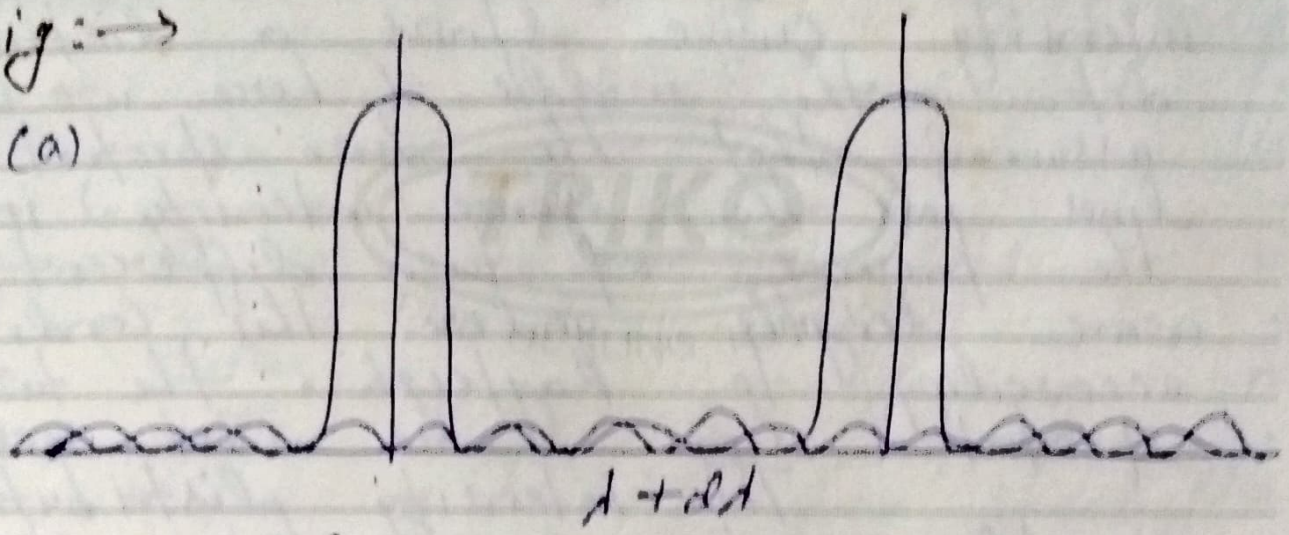
According to Lord Rayleigh's Criterion the two points sources are two spectral lines of equal intensity are just resolved by an optical instrument where the central maximum of the diffraction pattern due to one fall on the first minimum of the diffraction pattern of the other.

To illustrate the above criterion, consider the spectrum formed by a prism or grating. In the spectrum each line consists of central maximum having on either side alternate minima and maxima.

Let us consider the intensity distribution curves of two wave lengths λ and $\lambda + \delta\lambda$.

The separation between the central maximum will depend upon the value of d . If d is sufficiently large, the central maxima due to the two wave lengths are quite separate and the two spectral lines appear well resolved.

Fig: \rightarrow



However $d\lambda$ will have a limiting value for which the angular separation between their maxima is such that the central maxima of one coincides with the first minimum of one coincides with the first minimum of the other and vice versa.

In this case the resultant intensity curve shows a distinct dip in the middle (here we have assumed that the two spectral lines have the same intensity.) indicating the presence of two different wave lengths. Under this condition, according to Rayleigh, the two spectral lines are just resolved. The intensity distribution in the grating or prism spectra is of the form.

$$I = I_{\max} \frac{\sin^2 x}{x^2}$$

At the middle of the first minima $x = \pi$.
the intensity due to each is

given by, putting.

$$x = \frac{\pi}{2}$$

Hence, the total intensity at the middle at the maxima is given by,

$$I_{\text{middle}} = 2I_{\text{max}} \frac{\sin^2 \frac{\pi}{2}}{\left(\frac{\pi}{2}\right)^2} = \frac{8}{\pi^2} I_{\text{max}}$$

$$\therefore \frac{I_{\text{middle}}}{I_{\text{max}}} = \frac{8}{\pi^2} = 0.81 \text{ (approx).}$$

Thus, Rayleigh's Criterion may be stated as the two spectral lines are just resolved if the intensity at the dip in the middle is $8/\pi^2$ times the intensity at either of the maxima.

If however d is smaller than the two diffraction pattern will overlap and the resultant intensity

Curve shows as if there is only one maximum. Hence, in this case the two spectral lines are not resolved.